Investigate of Creep Response in Functionally Graded Material Rotating Disc with variable Thickness

Vandana Gupta
Department of Mathematics, Dashmesh Khalsa College, Zirakpur (Mohali), Punjab, India.
vaggarwal2584@gmail.com

Correspondence should be addressed to Vandana Gupta**:** vaggarwal2584@gmail.com

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Abstract – The objective of this paper to investigate analysis of creep in the Functionally Graded Material disc with variable thickness made of aluminum alloy-based metal matrix composite containing silicon carbide particles in presence of thermal gradients in the radial direction. It has been concluded that distributions of the stress and strain rate in an anisotropic disc got affected from the thermal gradients. Thus, the presence of thermal gradients in rotating disc plays a significant role in developing the creep response.

Keywords – Modeling, Composites, Creep, Functionally Graded Material, Thermal Gradients.

I. INTRODUCTION

Rotating disc is a vital component that uses in many engineering applications in the jet engine, turbine rotors, automotive breaks, flywheels, compressors and computer disc drives etc [1]. In all the engineering applications, the disc is subjected to operate at higher angular velocity and high thermal gradient [2-4]. Because of the high degree of influence on the properties of disc, creep analysis has gained the attention of the researchers [5-7]. Keeping this in mind, the study has carried out the analysis of creep behavior for the disc in presence/absence of thermal gradients [9-11].

II. ANALYSIS

The disc material is assumed to undergo steady state creep according to the threshold stress, as described by following Sherby's law [8].

$$\dot{\bar{\varepsilon}} = \left[M\left(\bar{\sigma} - \sigma_0\right)\right]^n \tag{1}$$

where $\dot{\bar{\varepsilon}}, M, \bar{\sigma}, \sigma_0, n$ are the effective strain rate, creep parameter, effective stress, threshold stress, the stress exponent.

The constitutive equations for creep are described under multiaxial stress, are expressed as,

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\left\{(G+H)\sigma_r - H\sigma_\theta\right\} \tag{2}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\left\{(H+F)\sigma_\theta - H\sigma_r\right\} \tag{3}$$

$$\dot{\varepsilon}_z = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\left\{-G\sigma_r - F\sigma_\theta\right\} \tag{4}$$

where the principal stresses are directed along (radial) r, (tangential) θ and (axial) z. F, G and H denote anisotropic constants of the material. $\dot{\bar{\varepsilon}}$ and $\bar{\sigma}$ denote the effective strain rate and the effective stress.

Dividing Eq. (2) by Eq. (3), we get

$$\phi(r) = \frac{d\dot{u}_r}{d\,r} \cdot \frac{r}{\dot{u}_r} \tag{5}$$

where, $x(r) = \frac{\sigma_r}{\sigma_\theta}$, is the ratio of radial and tangential stresses at any radius r and $\dot{u}_r = du/dt$ is the radial deformation rate. The equilibrium force in the radial's direction is,

$$\frac{d}{dr}\left(r\ h\ \sigma_r\right) - h\sigma_\theta + \rho\omega^2 r^2 h = 0 \tag{6}$$

Boundary Conditions are $\sigma_r\,(a) = 0 = \sigma_r\,(b)$ (7)

Integrating Eq. (5) with limit a to r on both sides,

$$\dot{u}_r = \dot{u}_{r_i}\ \exp\int_a^r \frac{\phi(r)}{r}\,dr \tag{8}$$

where, $\dot{u}_{r_i}$, is the radial deformation rate at the inner radius.

Dividing Eq. (8) by r and equated to Eq. (3),

$$\bar{\sigma} - \sigma_0 = \frac{(\dot{u}_{r_i})^{1/8}}{M}\,\psi(r) \tag{9}$$

where,

$$\psi(\mathrm{r}) = \left\{ \frac{\sqrt{\frac{\mathrm{G}}{\mathrm{F}} + \frac{\mathrm{H}}{\mathrm{F}}}}{r} \cdot \frac{\left[\left(\frac{\mathrm{H}}{\mathrm{F}} + \frac{\mathrm{G}}{\mathrm{F}}\right)x^2 - \frac{2\mathrm{H}\,x}{\mathrm{F}} + \left(1 + \frac{\mathrm{H}}{\mathrm{F}}\right)\right]^{1/2}}{\left[\left(1 + \frac{\mathrm{H}}{\mathrm{F}}\right) - \frac{\mathrm{H}}{\mathrm{F}}x + \frac{f_c - f_t}{\sigma_\theta}\right]}\ \frac{\dot{u}_r}{\dot{u}_{r_i}} \right\}^{1/8} \tag{10}$$

and

$$M\,(r) = e^{-35.38}\,P^{\,0.2077}\,T(r)^{\,4.98}\,V^{\,-0.622} \tag{11}$$

$$\sigma_0\,(r) = -0.03507P + 0.01057\,T(r) + 1.00536 - 2.11916 \tag{12}$$

Substituting $\dot{u}_r$ from Eq. (9) into Eq. (2) and simplify to get the tangential stress (σ_θ),

$$\sigma_\theta = \frac{(\dot{u}_{r_i})^{1/n}}{M}\ \psi_1\,(r) + \psi_2\,(r) \tag{13}$$

where,

$$\psi_1(r) = \frac{\psi(r)}{\left\{\left(\frac{F}{G+H}\right)\left[\left(\frac{G}{F} + \frac{H}{F}\right)x^2 - 2\,\frac{H}{F}\,x + \left(1 + \frac{H}{F}\right)\right]\right\}^{1/2}} \tag{14}$$

$$\psi_2(r) = \frac{\sigma_0}{\left\{\left(\frac{F}{G+H}\right)\left[\left(\frac{G}{F} + \frac{H}{F}\right)x^2 - 2\,\frac{H}{F}\,x + \left(1 + \frac{H}{F}\right)\right]\right\}^{1/2}} \tag{15}$$

The average tangential stress may be defined as

$$\sigma_{\theta_{avg}} = \frac{1}{b-a}\int_a^b \sigma_\theta\,dr \tag{16}$$

Now radial stress can be obtained by integrating Eq. (6) within limits a to b,

$$\sigma_r(r) = \frac{1}{r}\left[\int_a^r \sigma_\theta \, dr - \frac{\omega^2 \rho (r^3 - a^3)}{3}\right] \tag{17}$$

For a disc, the thermal conductivity can be calculated as,

$$K(r) = \frac{[100 - V(r)]K_m + V(r)K_d}{100} \tag{18}$$

Where $K_m = 247 W/m\,K$ is matrix conductivity and $K_d = 100 W/m\,K$ is diserpersoid conductivity. $T(r)$ is the temperature can expressed in termers of radius,

$$T(r) = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + a_5 r^5 \tag{19}$$

where the coefficients a_0, a_1, a_2, a_3, a_4 and a_5 for different disc are taken from Gupta et al.[4].

III. RESULTS AND DISCUSSION

For the purpose of computation, a computer program based on the analysis presented in this article has been developed to find stress and strain rates of the anisotropic FGM discs. For the analysis of creep behavior, some parameters have been taken for disc as shown in **Table 1**. The variation of tangential stresses in anisotropic rotating discs along the radius in the disc for presence and absence of thermal gradients has been shown in **Fig 1**. An anisotropic disc with thermal gradients has little higher the tangential near the inner radius and slightly lowers near the outer radius as compared the anisotropic disc without thermal gradients in **Fig 2.** The variation of radial stresses in anisotropic rotating discs along the radius in the disc for presence and absence of thermal gradients has been shown in **Fig 3**. The radial stresses are not much affected by introducing thermal gradients in the anisotropic disc as the values of radial stress are very close for presence and absence of thermal gradients. The variation of tangential strain rate in anisotropic rotating discs along the radius in the disc for presence and absence of thermal gradients has been shown in figure 3. The magnitude due to thermal gradients is smaller compared to disc without thermal gradients. But the trend of variations of tensile strain rate remains the same in both discs. The variation of radial strain rate in anisotropic rotating discs along the radius in the disc for presence and absence of thermal gradients has been shown in **Fig 4**. The magnitude of radial strain rate is smaller in an anisotropic disc with thermal gradients compared to disc without thermal gradients.

Density of disc material $\rho = 2812.4\, kg/m^3$

Inner radius of disc, $a = 31.75 mm$

Outer radius of disc, $b = 152.4 mm$

Particle size, $P = 1.7\, \mu m$

Particle content, $V = 20\%$

Operating conditions:

Angular velocity of Disc, $\omega = 15{,}000\, rpm$

Operating temperature, $T = 623\, K$

Creep duration, $t = 180\, hrs$

The values of coefficients operating under thermal gradients, $a_0 = 619.69, a_1 = 0.6083,$ $a_2 = -0.0208, a_3 = 3.27 \times 10^{-4},$ $a_4 = -1.96 \times 10^{-6}, a_5 = 4.43 \times 10^{-9}$

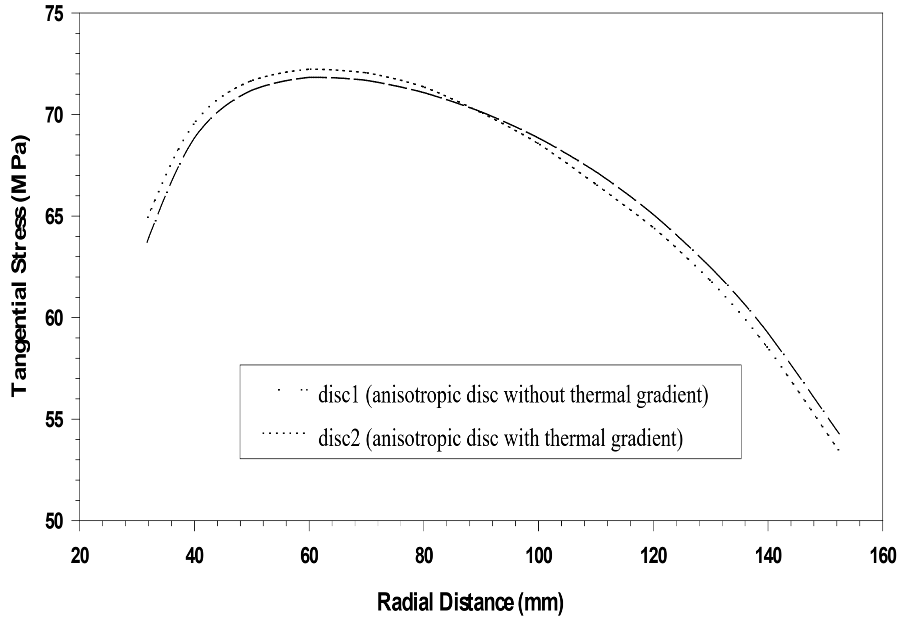

Fig 1. Variation of Tangential Stresses in Anisotropic Disc With/Without Thermal Gradients.

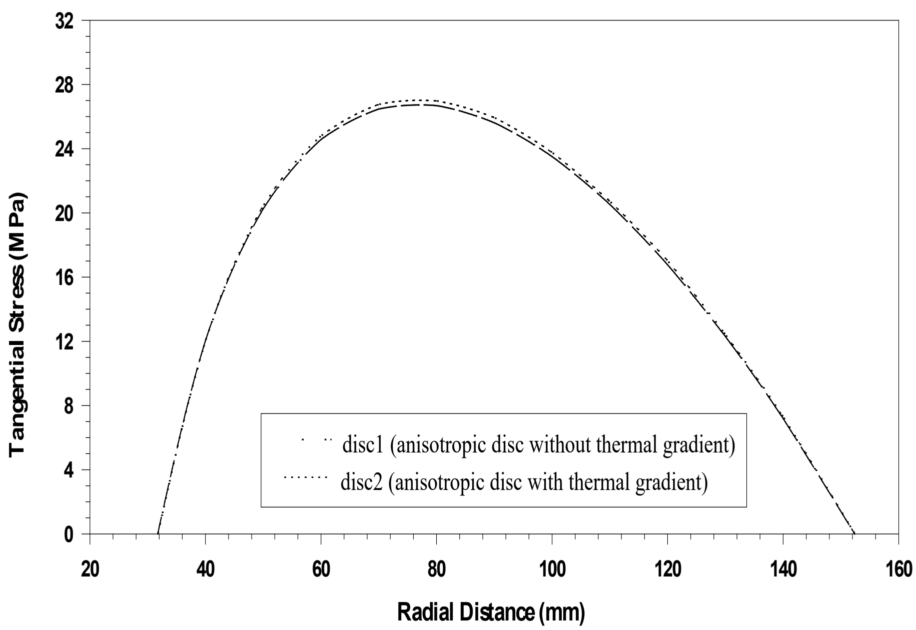

Fig 2. Variation of Radial Stresses in Anisotropic Disc With/Without Thermal Gradients.

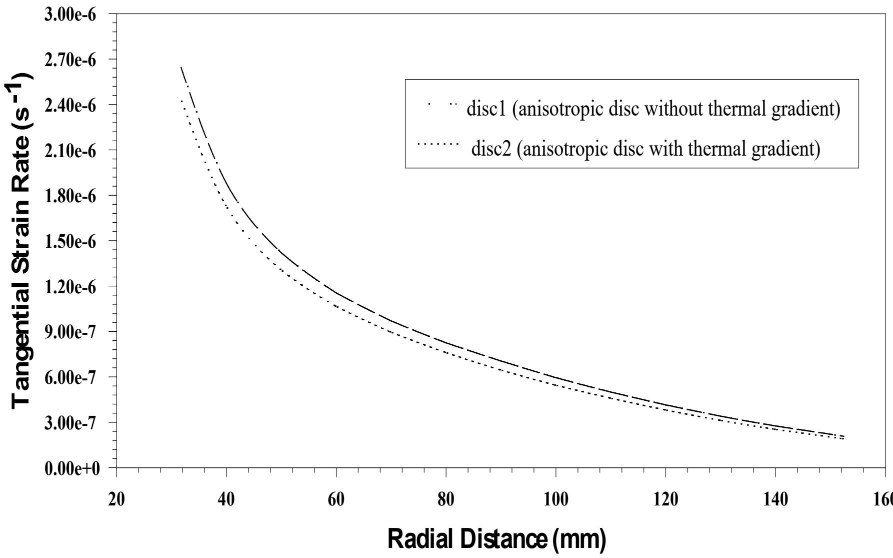

Fig 3. Variation of Tangential Strain Rates in Anisotropic Disc With/Without Thermal Gradients.

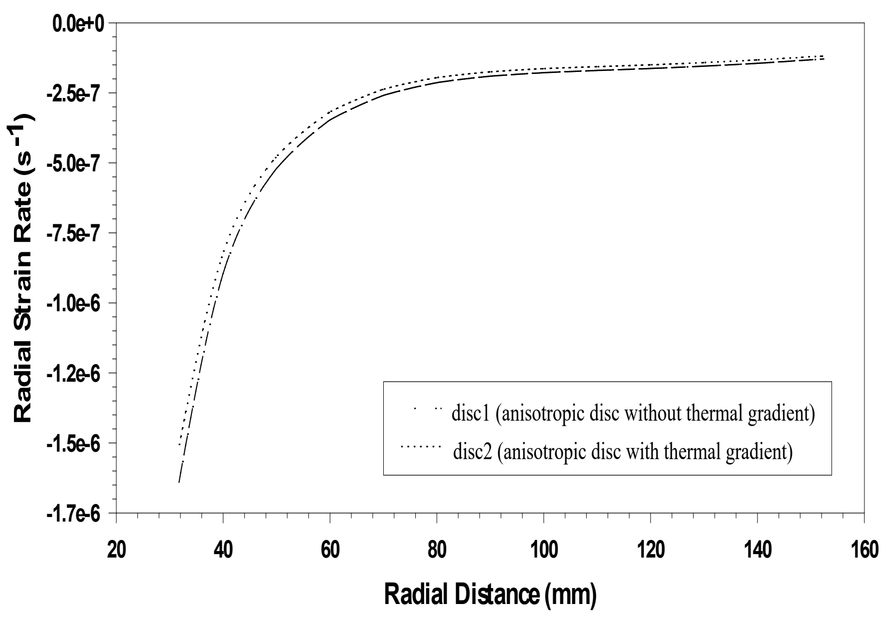

Fig 4. Variation of Radial Strain Rates in Anisotropic Disc With/Without Thermal Gradients.

IV. CONCLUSION

The distribution of stresses and strain rates in a rotating Al-SiCp disc are significantly affected by the thermal gradients although its effect on the distribution of strain rate is relatively higher than stresses. So, this aspect which is requiring for safe designing of a rotating disc should be taken care of.

Data Availability
No data were used to support this study.

Conflicts of Interest
The author(s) declare(s) that they have no conflicts of interest

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